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Comment on "Interception of High-Speed Target by Beam Rider Missile"

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IN Ref 1, it is possible to express the missile coordinates explicitly in terms of the normal elliptic integrals F and E

The linear differential equation considered by the authors was

$$(d/d\gamma)(\cot\theta) + \tfrac{1}{2}\cot\gamma\cot\theta + \tfrac{1}{2} = 0 \tag{1}$$

Let ds_m be an element of arc of the missile trajectory and

$$ds_m = V_m dt = (V_m/V_t)V_t dt = \tau R d(\cot\theta)$$

With $\tau R = c$ and $\cot\theta = x_m/y_m$, we have,

$$ds_m = cd\left(\frac{x_m}{y_m}\right) = c\left(\frac{dx_m}{y_m} - \frac{x_m dy_m}{y_m{}^2}\right) \tag{2}$$

Since

$$dx_m = \cos\gamma ds_m \qquad dy_m = \sin\gamma\, ds_m$$

Eq (2) becomes

$$ds_m = c\left(\frac{\cos\gamma}{y_n} - \frac{x_m\sin\gamma}{y_m{}^2}\right)ds_m \tag{3}$$

Therefore,

$$y_m{}^2 = c(y_m\cos\gamma - x_m\sin\gamma) \tag{4}$$

This equation clearly shows that the velocity of the missile is tangent to the circle centered at the origin and of radius $y_m{}^2/c$, a result mentioned in Ref 2 using different arguments

Therefore,

$$\cot\theta = \frac{x_m}{y_m} = \cot\gamma - \frac{y_m}{c\sin\gamma} \tag{5}$$

Using Eq (5) in (1), one obtains the differential equation

$$2\sin\gamma(dy/d\gamma) - \cos\gamma y + 1 = 0 \tag{6}$$

which is also linear with $y = y_m/c$

Integrating Eq (6) yields

$$y = \cos\phi\left(C + \frac{F(\phi,k) - 2E(\phi,k)}{2^{1/2}}\right) + \sin\phi(1 + \cos^2\phi)^{1/2} \tag{7}$$

and using (4) we obtain the x coordinate as

$$x = -\left(C + \frac{F - 2E}{2^{1/2}}\right) \times \left[C + \frac{F - 2E}{2^{1/2}} + \tan\phi(1 + \cos^2\phi)^{1/2}\right] \tag{8}$$

where F and E are normal elliptic integrals of the first and second kind with moduli $k = 1/2^{1/2}$ and arguments $\phi = \text{arc}\cos(\sin\gamma)^{1/2}$

The constant C is determined by the initial conditions We also have

$$\cot\theta = \frac{x}{y} = -\frac{1}{\cos\phi}\left(C + \frac{F - 2E}{2^{1/2}}\right) \tag{9}$$

which is Eq (9) in Ref 1

Using the initial conditions, when $x = y = 0$, $\theta = \theta_0 = \gamma_0$, we have for the constant C,

$$C = \frac{2E(\phi_0) - F(\phi_0)}{2^{1/2}} - \tan\phi_0(1 + \cos^2\phi_0)^{1/2} \tag{10}$$

By these expressions it can easily be seen that:

1) $y = 1$ is an asymptote since x becomes infinite for $\gamma = 0$ except when

$$C - \frac{2E(\pi/2) - F(\pi/2)}{2^{1/2}} = 0$$

or $C = 0\ 599$ This gives $\theta_0 = 37°$

2) The points where the velocity is directed vertically are such that $\gamma = \pi/2$, $\phi = 0$ Therefore $x = -C^2$ and $y = C$

Hence, they are situated on the parabola $y^2 = -x$, a result also mentioned in Ref 2 using different arguments

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"Equilibrium" Gas Composition Computation with Constraints

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IT has been suggested in a recent note[1] that one may "freeze" a particular species in a chemical equilibrium computation by introducing a fictitious second set of species and manipulation of the real and imaginary sets of species, within the usual procedures of the computation routine The purpose of this note is to point out that such a constraint may be imposed on the equilibrium calculation in another manner when one is using the popular minimization of free energy technique [2] In this approach the Gibbs free energy is minimized subject to the constraints of the mass balances

$$\sum_{i=1}^{n} a_{ji}x_i = b_j \tag{1}$$

where a_{ji} is the number of atoms of element j in species i, x_i is the moles of species i per unit mass, and b_j is the number of gram-atoms of element j per unit mass

It is possible, however, to impose *a priori* relations (constraints) among the x_i composition variables by considering the constraints to be pseudo elements, as long as the relations are of the form of Eq (1), i e, linear Thus the formula matrix a_{ji} would have a column for each species and m rows for each of the m real chemical elements, as usual, with an additional row for each constraint Possible constraints

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would be freezing of a given species (x_i = constant), two species having a given ratio, or whatever other information or prejudices one has about linear relations among the x_i This approach could be extended to the use of inequalities by using a nonlinear optimization routine[3] to minimize the Gibbs free energy, subject to linear equality or inequality constraints

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[1] Bahn, G S, "Thermodynamic calculation of partly frozen flows," AIAA J 1, 1960–1961 (1963)

[2] White, W B, Johnson S M, and Dantzig, G B, "Chemical equilibrium in complex mixtures," J Chem Phys 28, 751–755 (1958)

[3] Rosen, J B, "The gradient projection method for nonlinear programming Part I Linear constraints," J Soc Ind Appl Math 8, 181–217 (1960); a program in FAP language for this approach is available as SHARE No 1399

Comments on "Wing-Tail Interference as a Cause of 'Magnus' Effects on a Finned Missile"

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SINCE the publication of Ref 1, the present authors have had several private communications about their differences concerning the magnus effects on a finned missile[2] We find that our views actually are not too far different, and we have been able to reach agreement on the major issues The following is being published in order that the picture of the aerodynamics of a rotating wing will be clarified

1) The angle of attack and the lift distribution on a rotating wing have spanwise variations that are dependent on the rotation helix angle $\omega r/U$

2) The integrated lift on the wing and the lift distribution can be obtained from

$$L = \int_a^{a+s_0} qC_{L\alpha}\left(\delta - \frac{r\omega}{U}\right) C(r)dr$$

where

q = dynamic pressure
$C_{L\alpha}$ = stationary wing lift curve slope
δ = wing deflection angle
r = spanwise distance from the center of rotation
ω = rate of rotation of the wing
U = forward velocity of the wing
C = wing chord $C = C(r)$ which is dependent on the wing geometry
a = distance from the rotation centerline to root chord
s_0 = distance between root chord and tip chord

3) The exact integrated lift and lift distribution cannot be determined until the wing geometry is fixed When the wing is in a free-spin condition, the net rolling moment is

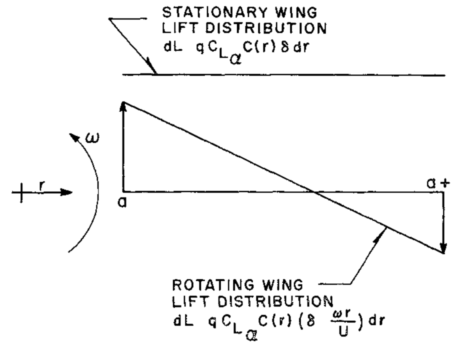

Fig 1 Comparison of the lift distribution on a stationary and a rotating wing

zero, $\int rdL = 0$, and the integrated lift can be shown to be small for any conventional wing (10 to 20% of the stationary lift) Also, the spanwise lift distribution can be approximated and is compared with the stationary wing lift distribution in Fig 1 If only the integrated lift is considered in the problem, then the rotation effect is small If the lift distribution along the span must be considered, then the rotation effects can be of prime importance

4) The change in the lift distribution due to rotation will alter the wake pattern aft of the wing and will change the wing-tail interference factors accordingly Also, the wing-tail interference factors are a function of the tail position in the wing wake The assumptions of $\eta_d = 0$ and $\eta_b = 1$ [Eqs (31) and (32) of Ref 2] apparently work well for the case considered but may not work for the general case

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Comment on "A Theoretical Interaction Equation for the Buckling of Circular Shells under Axial Compression and External Pressure"

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IN his note, Sharman[1] stated that he assumed $m = 1$ in the computations and reasoned that "assuming $m = 1$ restricts the valid solutions to positive external pressure only" The author believes that this reasoning is slightly erroneous, since it implies a jump in m at $(p/p_0) = 0$ Sharman's reasoning demands that, as $(p/p_0) \rightarrow 0$, m for minimum σ_c remains unity, until at $(p/p_0) = 0$, the well-known case of pure axial compression, $m \gg 1$ Hence at $(p/p_0) = 0$ there are two possible configurations, one with $m = 1$ and one with usual $m \gg 1$, which seems unlikely

For clarification, some points were calculated (with similar parameters as in Ref 1) near the R_c axis and the calculations indicated that there is indeed a narrow transition region, and

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